J. K. SHAH CLASSES
SYJC - MATHS TEST

BRANCH : CHARNIROAD | **MARKS : 40**
SUB : MATHS | **Time : 1 Hour 30 min.**
Date : 09 / 10 /2016

SOLUTION

Ans.1. (A) Attempt any SIX of one following **(12 Marks)**

(1) Obtain the Crude Death Rate (CDR) for no. of persons from the date given below :

AGE GROUP	No. of Person (In'000)	No. of Deaths
Below 10	12	150
10-30	20	110
30-45	35	380
45-70	24	210
Above 70	14	540
	105	**1390**

$\Sigma p_i = 105 \times 1000$

$\Sigma D_i = 1390$

$$C.D.R = \frac{1390}{1,05,000} \times 1,000$$

C.D.R = 13.11 per thousand.

Ans.(2) n=100, $\bar{x}$ = 62, $\bar{y}$ 53, $\sigma x = 10$, $\sigma y = 12$, $\Sigma (x - \bar{x})(y - \bar{y}) = 8000$

Karl pearsons coefficient correlation in given by

$$r = \frac{\frac{1}{n}[\Sigma (xy) - \bar{x}.\bar{y}]}{\sqrt{\frac{1}{n}\Sigma x^2 - x^2}\ \sqrt{\frac{1}{n}\Sigma y^2 - y^2}}$$

$$= \frac{\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}}{\sigma x.\ \sigma y}$$

$$= \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n\sigma x.\ \sigma y}$$

$$= \frac{8,000}{100 \times 10 \times 12} = \frac{8}{12}$$

= 0.67

Ans.(3) Capital = Same

Time period.

Ameena = 12 months

Yasmin = 9 months

Shobana = 5 months

Profit is Directly portional to ratio of capital

Time period ratio = 12 : 9 : 5

Profit = 23,400

Ameena's Share = $23,400 \times \frac{12}{26}$ = ₹ 10,800

Yasmin's Share = $23,400 \times \frac{9}{26}$ = ₹ 8,100

Shobana Share = 23,400 x $\frac{5}{26}$ = ₹ 4,500

Ans.(4) $x + 2y \geq 4$; $2x - y \leq 6$

Corresponding Equalities :-

$x + 2y = 4$

$2x - y = 6$

Point on intersection :- A(4, 0) C-(3, 0)

B(0, - 2) D(0,- 6)

Origin Test :- $x + 2y \geq 4$

$0 + 0 \geq 4$

$0 \geq 4$

False .....(i)

$2x - y \leq 6$

$0 - 0 \leq 6$

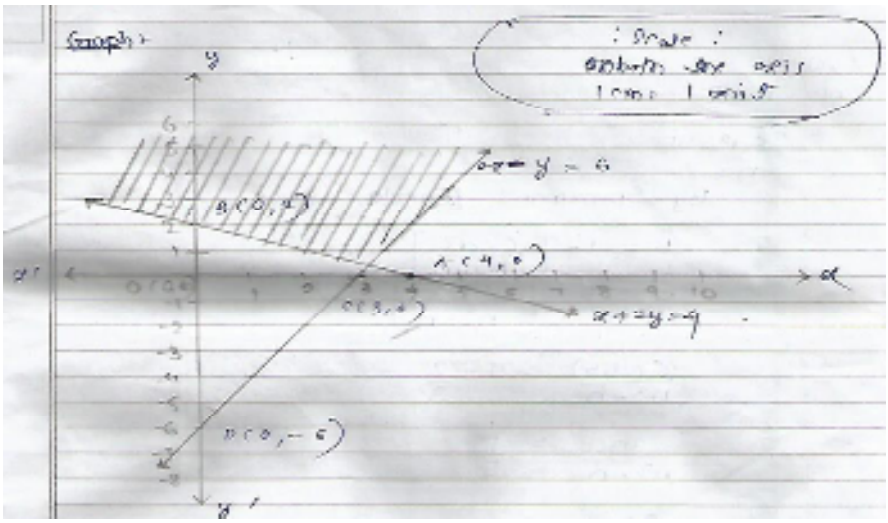
$0 \leq 6$

True .....(ii)

Shading :- (i) Away from the origin

(ii) Towards the origin

Graph :-



The shaded portion represents the graphical solution for the given system of linear Ineguations.

Ans.(5) **R** = $\frac{2}{3}$

Σd^2 = 55

N - ?

By Spearman's formula,

$$R = 1 - \left[\frac{6\Sigma d^2}{n^3 - n}\right]$$

$$\frac{2}{3} = 1 - \left[\frac{6\,(55)}{n^3 - n}\right]$$

$$\frac{6\,(55)}{n^3 - n} = 1 - \frac{2}{3}$$

$$\frac{330}{n^3 - n} = \frac{1}{3}$$

$990 = n^3 - n$

$n^3 - n = 1{,}000 - 10$

$\boxed{n = 10}$

Ans.(6) Given : $l_4 = 60$ and $L_4 = 45$, $P_4 = ?$

We know that

$$l_4 = \frac{l4 + l5}{2}$$

$$45 = \frac{60 + l5}{2}$$

$$\therefore \; l_5 = 30$$

Now, we have

$$\boxed{d_4 = l_4 - l_5}$$

$$\therefore \; d_4 = 60 - 30$$

$$\therefore \; d_4 = 30$$

Finally, $\boxed{P_x = 1 - q_x}$

$$P_4 = 1 - q_4$$

$$= 1 - \left[\frac{d4}{l4}\right]$$

$$= 1 - \left[\frac{30}{60}\right]$$

$$P_4 = 1 - 0.5$$

$$\boxed{P_4 = 0.5}$$

Ans.(7)

	1 st Cycle	**2nd Cycle**
Ratio of Cycle	16	23
Let the common multiple be 'X'	16x	23x
Price increased by	16x (10%) $16x \left(\frac{10}{100}\right)$ $\frac{16x}{10}$	₹ 477
New price	$16x + \frac{16x}{10}$ $\frac{716x}{10}$	[23x + 477]
New price ratio	**11 :**	**20**

As we have,

$$\frac{\frac{176x}{10}}{23x + 477} = \frac{11}{20}$$

$$\therefore \; 20\left(\frac{176x}{10}\right) = 11(23x + 477)$$

$$\therefore \; 2(176)x = 11(23x + 477)$$

$$\therefore \; 2\left(\frac{176x}{11}\right) = 23x + 477$$

$$\therefore \; 2(16x) = 23x + 477$$

$$\therefore \; 32x - 23x = 477$$

$$\therefore \; 9x = 477$$

$$\therefore \; x = 53$$

Hence original price of 1st Cycle

= 16x

= 16(53)

= 848

Original price of Cycle 2

= 23x

= 23(53)

= 1219

Ans.(8) 3x + 2y = 26 .....(1)

6x + y = 31 ......(2)

Solving eq (1) & (2) simultaneously.

Multiplying Σq (1) by2

6x + 4y = 52

6x + y = 31

\- -

3y = 21

y = 7

∴ x = 4

Let the regression equation x on y be

6x + y - 31 = 0 6x + y = 31

bxy = $\frac{-b}{a}$ = $\frac{-1}{6}$

Let y on x be

3x + 2y = 26

byx = = $\frac{-a}{b}$ = $\frac{-3}{2}$

r^2 = bxy x byx

r^2 = $\frac{-1}{6}$ = $\frac{-3}{2}$ = $\frac{3}{12}$

= $\frac{1}{4}$

Since $r^2 \leq 1$ our assumption is correct

r < 1

$\boxed{r = \sqrt{bxy \cdot byx}}$

= $\sqrt{\frac{-1}{0}, \frac{-3}{2}}$

$\boxed{r = \frac{-1}{2}}$

Since, byx Σ bxy are negative

Correlation Coefficient of regression $\frac{-1}{2}$

Ans2 (A) Attempt any Two of the following : **(6 Marks)**

(1) Optional Sequence of books can be obtained as Follows:

I	III	V	IV	II

Now, Min^{m} elapsed time can be Computed as follows :

MACHINE / BOOK SEQUENCE	Printing : M_1		Binding : M_2		Ideal time for M_2
	Time IN	Time OUT	Time IN	Time OUT	
I	0	3	3	9	3
III	3	7	9	16	0
V	7	14	16	20	0
IV	14	19	20	23	3
II	19	26	26	28	0
	Total Ideal time for M_2 ⟹				**6**

From above tabular format :

(A) Total elapsed time (T) = 28 HRS

(B) Ideal time for M_1 :

$= T - \left(\begin{matrix}\text{Total time taken by Machine } M_1 \text{ to} \\ \text{process all the jobs.}\end{matrix}\right)$

$= 28 - 26$

$\therefore$ Ideal time for 'H_1' = 2 HRS

(c) Ideal time for 'H_2' = 6 HRS

Ans.(2) Capital – Different

OM = 30,000

JAI = 40,000

JAGDISH = 50,000

Time period - Different

OM = 12 months

JAI = 6 months

JAGDISH = 3 months

Profit x (directly proportional) to the product of Capital & Time Period.

Profit ratio = (30,000 x 12) : (40,000 x 6) : (50,000 x 3)

= 3,60,000 : 2,40,000 : 1,50,000

Profit ratio = 12 : 8 : 5

Om Share = 17,500 = $\frac{12}{25}$ = 700 x 12 , = ₹ 8,400

Jai Share = 17,500 = $\frac{8}{25}$ = 700 x 8, = ₹ 5,600

Jagdish Share = 17,500 = $\frac{5}{25}$ = 700 x 8, = ₹ 3,500

Ans.(3) Given x (height of Male)

y (Weight of Male)

The class interval of x

150 – 154; 155 – 159; 160 – 164; 165 – 169

Class interval of y

60 – 64; 65 – 69; 70 – 74

We shall take the class Interval of X along 1st row Σ for y along 1 column for the Following bivariate frequency distribution Table.

Bivariate frequency distribution table of height and weight

Wgh / hgh	150 - 154	155 - 159	160 - 164	165 - 169	Total
60 – 64	11 (2)	---	11 (2)	----	4
65 – 69	1 (1)	111 (3)	1 (1)	----	5
70 – 77	11 (2)	11 (2)	1111 (4)	111 (3)	11
Total	5	5	7	3	20

Marginal Distribution:

For 'x'

Height of Male	150 – 154	155 – 159	160 – 164	165 – 169	Total
No. of Male	5	5	7	3	20

For 'Y'

Weight (y)	60 – 64	65 – 69	70 – 74	Total
No. of Male	4	5	11	20

Conditional frequency Distribution

For x when y in the class interval 60 - 64

Height (x)	150 – 154	155 – 159	160 – 164	165 – 169	Total
No. of Male	2	0	2	0	4

(B) Attempt any Two of the following : (8 Marks)

Ans.(1) $X + y \leq 5$ $X + y \leq 8$ $4x + 3y \geq 12$

A (5, 0) C (8, 0) E (3, 0)

B (0 ,5) D (0, 4) F (0, 4)

Origin Test - $X + y \leq 5$

$0 + 0 \leq 5$

True

$X + 2y \leq 8$

$0 + 0 \leq 8$

True

$4x + 3y \geq 12$

$0 + 0 \geq 12$

FALSE

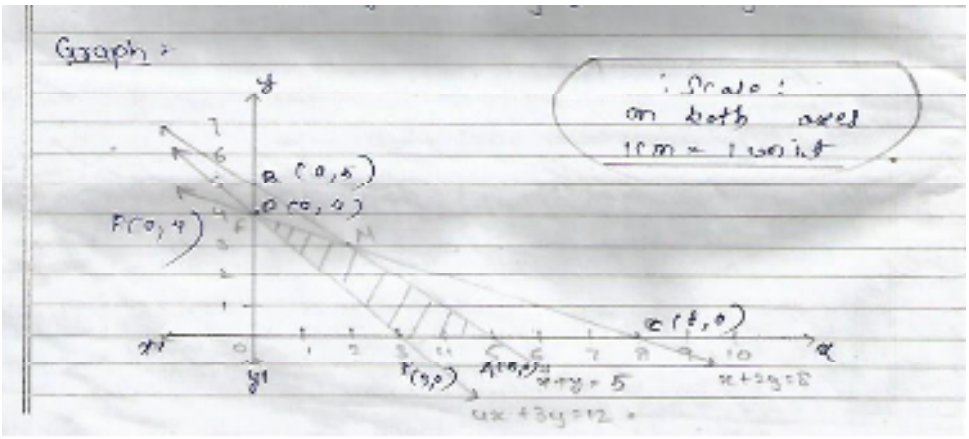
Shading :-

$x + y = 5$ - Towards the origin

$x + 2y = 8$ - Towards the origin

$4x + 3y = 12$ - Away from the origin

Graph :-



The region bounded by EAMD is feasible region

To get cordinates of M

$x + y = 5$ (1) , $x + 2y = 8$

Subtracting eg (1) from (2)

$x + 2y = 8$

$x + y = 5$

$- \quad -$

$y = 3$

$\boxed{y = 3}$

Substitution $y = 3$ in eq (1)

$x = 2$

$\therefore$ M (2,3)

Points	z = 2x-y (Minimize)
E (3,0)	z = 2(3) – 9 = 6
A (5,0)	z = 2(5) – 0 = 10
M (2,3)	z = 2(2) – 3 = 1
D (0,4)	z = 2 (0) – 4 = -4

$\therefore$ z is minimum at D (0,4) and the value is -4.

Ans.(2) No. of pair of observations (n) = 10

Sum of x – Series (Σx) = 9

Sum of y – Series (Σy) = 5

Sum of square of x – Series(Σx^2) = 653

Sum of Square of y - Series (Σy^2) = 595

Sum of product of x and y series (Σxy) = 534

X and y Series

$$r = \frac{n\Sigma xy - \Sigma x.\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{10(534) - (9)(5)}{\sqrt{10(653) - (9)^2 - (\Sigma x)^2}\ \sqrt{10(595) - (5)^2}}$$

$$= \frac{10(534) - 45}{\sqrt{6530 - 81}\ \sqrt{5950 - 25}}$$

$$= \frac{5340 - 45}{\sqrt{6449}\ .\ \sqrt{5925}}$$

$$= \frac{5295}{80.3\ x\ 76.9}$$

$$= \frac{5295}{6175}$$

r = 0.85

Hence, Product moment correlation coefficient.

is = 0.85

	A	B	C
Exp	90%	80%	70%
Saving	10%	20%	30%
Saving Ratio	3	4	7

Part (1)

$$\frac{Saving\ (A)}{Saving\ (B)} = \frac{3}{4}$$

$$\frac{90\ (Exp\ of\ A's\ Salary)}{80\ (Exp\ of\ B's\ Salary)} = \frac{3}{4}$$

$$\frac{(Exp\ of\ A's\ Salary)}{(Exp\ of\ B's\ Salary)} = \frac{3}{4}$$

Part (2)

$$\frac{Saving\ (B)}{Saving\ (C)} = \frac{4}{7}$$

$$\frac{20}{30}\ \frac{(Exp\ of\ B's\ Salary)}{(Exp\ of\ C's\ Salary)} = \frac{4}{7}$$

$$\frac{Exp\ of\ B\ Salary}{Exp\ of\ C\ Salary} = \frac{4}{7}\ x\ \frac{3}{2} = \frac{6}{7}$$

Now, savings of A , B & C are in ratio.

A : B : C

3 2

6 : 7

9 : 6 : 7

Total Salary is 66,000

∴ A's Salary = 66,000 x $\frac{9}{22}$ = 27,000

B's Salary = 66,000 x $\frac{6}{22}$ = 18,000

C's Salary = 66,000 x $\frac{7}{22}$ = 21,000

Ans.3 (A) **Attempt any Two of the following :** **(6 Marks)**

(1) n = 6

$\bar{x} = \frac{\Sigma x}{n} = \frac{21}{6} = \frac{7}{2}$ - 3.5

$\bar{y} = \frac{\Sigma y}{n} = \frac{30}{0} = 5$

	X	Y	x.y	x^2	y^2
	1	2	2	1	4
	2	4	8	4	16
	3	7	21	9	49
	4	6	10	16	36
	5	5	10	25	25
	6	6	36	36	36
Total	**21**	**30**	**116**	**91**	**166**

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\ \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{6(116) - (21)(30)}{6(91) - (21)^2}$$

$$byx = \frac{60}{105} = 0.6285$$

$$(y - \bar{y}) = byx\ (x - \bar{x})$$

$$y - 30 = 0.6285\ (x - 21)$$

$y = 0.6285x + 2.80025$ ....Required eg^n of regression line y on x.

for $x = 10$

$y = 0.6285\ (10) + 280025$

$= 6.285 + 2.80025$

$\boxed{Y = 9.08525}$

Ans.(2)

X	Lx	Axlx-ux-1	X=dx/Lx	Px=(1-Lx)	$Lx+\frac{Lx+Lx+1}{2}$	Tx= Σ L.x	ex^o=Tx/Lx
0	1000	40	0.04	0.96	980	2880	2.88
1	960	160	0.17	0.83	880	1900	1.979
2	800	200	0.25	0.75	700	1020	1.275
3	600	580	0.97	0.03	310	320	0.533
4	20	20	1	0	10	10	0.50
5	0	-	-	-	-	-	-

Ans.(3)

	Mr. Ahuja	Mr. Sinha
Capital	75,000	5,000
Time	12 month	5 months
Capital	5,000	40,000
Time	7 months	7 months

Rule (3)

Profit is directly proportional to the product of capital

[75,000(12)+5,000(7)] : [5,000(5)+40,000(7)]

5,000[15(12)+(7)] : 5,000 [10(5) + 8,000(7)]

187 : 106

A's share of Profit = $11720 \times \frac{187}{293}$

$= 40 \times 187$

$= ₹\ 7480$

B's Share of Profit = $11720 \times \frac{106}{293}$

$= 40 \times 106$

$= ₹\ 4240$

(B) Attempt any Two of the following : (8 Marks)

Ans.(1)

	Old Alloy Zn = 37% Sn = 63%	Zinc Added Let x kg is added	New Alloy ZA=70% Sn=30%
Weight	400	x	400 + x
Weight of Z	$400 \times \frac{37}{100}$ = 148	x	$(400 + x)\left(\frac{7}{10}\right)$

Since x is the weight of the zinc added to old alloy so that new alloy consist of 70% of Zinc

Weight of zinc in oOld alloy + weight of zinc added = weight of zinc in new alloy

$148 + x = 400 + x\left(\frac{7}{10}\right)$

$\frac{148+X}{7} = \frac{400+x}{10}$

$1480 + 10x = 2,800 + 7x$

$3x = 1320$

$x = \frac{1320}{3}$

$x = 440kg$

$\therefore$ 440 kg of Zinc to be added.

Ans.(2) Compute rank correlation coefficient for the marks in History and marks in Geography obtained by **8** Students.

HISTORY	Geog	R_1	R_2	CR_1	CR_2	$D=1(CR_1- CR_2)$	d_2
70	80	1	1	1.5	1.5	0	0
70	60	1	5	1.5	5	3.5	12.25
65	80	3	1	3	1.5	1.5	2.25
60	70	4	3	4	3	1	1
55	65	5	4	5	4	1	1
50	50	6	6	6	6	0	0
40	42	7	7	7	7	0	0
30	28	8	8	8	8	0	0
	n = 8						**Σd^2 = 16.50**

T.C.F

	Rank	M	TCF = ½(m^2= m)
T_1	1	2	0.5
T_2	1	2	0.5

By spearmans Rank Correlation of coefficient

$$R = 1- \left[\frac{6(\Sigma d^2+ T_1+ T_2)}{n^3-n}\right]$$

$$= 1- \left[\frac{6(16.50 + 0.5 + 0.5)}{512-8}\right]$$

$$= 1- \left[\frac{6(17.5)}{504}\right]$$

$$= \frac{504-505.0}{504}$$

$$= \frac{\cancel{399}\ 133}{\cancel{504}\ 168} = 0.7917$$

$\therefore$ R = 0.7917

Ans.3

$5x + y \geq 10$

$2x + 2y \geq 12$

$x + 4y \geq 12$

The objective fun z = 3x + 2y is to be minimized

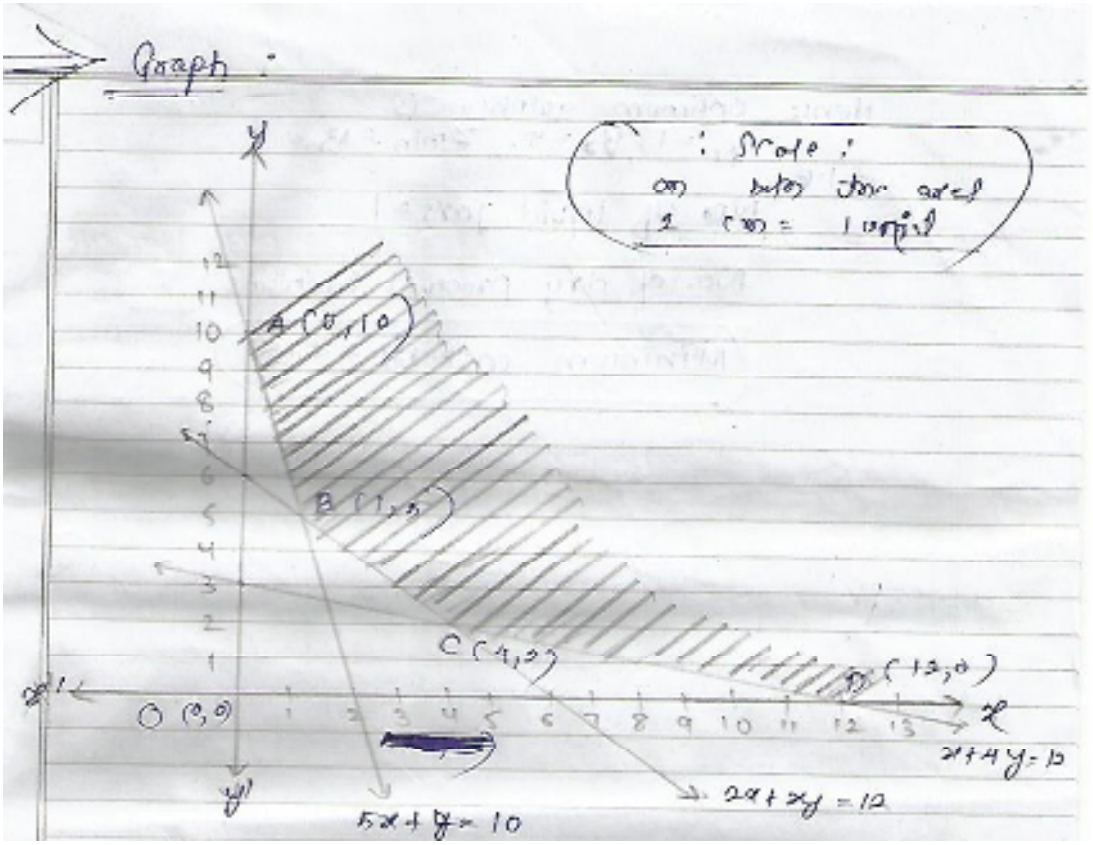
Now, 5x + y = 10

Given in equality	5x + y ≥ 10	2x + 2y ≥ 12	x + 4y ≥ 12
Corresponding equality	5x + y = 10	2x + 2y = 12	x + 4y = 12
P.O.I	(A) (2, 0)	(C) (6,0)	(E) (12,0)
	(B) (0,10)	(D) (0,6)	(F) (0,3)
Origin Test :	5x + y ≥ 10	2x + 2y ≥ 12	x + 4y ≥ 12
	0 ≥ = 10	0 ≥ = 12	0 ≥ = 12

Shading: $0 \geq = 10$ Towards the origin

$0 \geq = 12$ Towards the origin

$0 \geq = 12$ Towards the origin



From graph ABCD is unbounded feasible region. This is a convex polygon whose lower verticals are A(,10) B(1,5) C(4,2), D(12,0). At least one of the vertices the value of objective function. $Z = 3x + 2y$ will be minimum.

At A(0,10)	$z = 3(0) + 2(10) = 20$
B (1,5)	$z = 3(1) + 2(5) = 13$
C (4,2)	$z = 3(4) + 2(2) = 16$
D(12,0)	$z = 3(12) + 2(0) = 36$

At B(1,5) the value of Z is minimum

Hence, optimum solution is

$X = 1, Y = 5, Z \text{ min} = 13$

i.e No. off liquid Jar = 1

No. of dry product cartoons = 5

(Minimum cost ₹ 13.)